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Systematic Errors in Stereo PIV When Imaging through a Glass Window

Richard Green and Kenneth W. McAlister

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SUMMARY

This document assesses the magnitude of velocity measurement errors that may arise when performing stereo PIV with the cameras viewing through a thick, refractive window and where the calibration is performed in one plane only. The effect of the window is to introduce a refractive error that increases with window thickness and the camera angle of incidence. The calibration should be performed while viewing through the test section window; otherwise, a potentially significant error may be introduced that affects each velocity component differently. However, even when the calibration is performed correctly, another error may arise during the stereo reconstruction if the perspective angle determined for each camera does not account for the displacement of the light rays as they refract through the thick window. Care should be exercised when applying a single-plane calibration since certain implicit assumptions may, in fact, require conditions that are extremely difficult to meet in a practical laboratory environment. It is suggested that the effort expended to ensure this accuracy may be better expended in performing a more lengthy volumetric calibration procedure, which does not rely upon the assumptions implicit in the single-plane method and avoids the need for the perspective angle to be calculated.

NOMENCLATURE

a	distance from object plane (along normal) to center of lens (mm)
c	length of object field along x (mm)
d_i	distance between lens and image plane, measured along lens axis (mm)
d_o	distance between lens and object plane, measured along lens axis (mm)
f	focal length of lens (mm)

H	object height (mm)
h	image height (mm)
i	angle of incident light ray (deg)
l	distance between lens and object centers (mm)
M	magnification
n	index of refraction
p	arbitrary point along object plane (mm)
q	coordinate normal to lens plane (mm)
R	distance between center of lens and Scheimpflug coincident point (deg)
r	angle of refracted light ray (deg)
t	window thickness (mm)
X, Y	coordinates in image plane (pixels)
x, y	coordinates in object plane (mm)
z	coordinate normal to object plane (mm)
α_i	angle between image and lens planes (deg)
δ	object plane distance (mm)
ϵ	perspective error (mm)
θ	perspective angle in x-z plane (deg)
θ_o	angle between object and lens planes (deg)
λ	coordinate in lens plane (mm)
σ	locations near lens measured along x (mm)

Superscripts

$'$	perspective measurement
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Subscripts

c	geometric center of object
i	position on image plane and coincident with the lens axis
im	image
L	left position, or reference to an arbitrary perspective view
N	normal to window
o	position on object plane and coincident with the lens axis

<i>obj</i>	object
<i>R</i>	right position, or reference to an arbitrary perspective view
<i>r</i>	refracted

Abbreviations

PIV	particle image velocimetry
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INTRODUCTION

Errors in PIV are often categorized as either random or bias errors. The random errors are often consequences of aspects of the experiment that are difficult to control and contribute to a general uncertainty in the result, while bias errors are systematic and lead to a degree of over or under estimation of the velocity. Bias errors are the result of the characteristics of the illumination, particle size, image recording equipment, and image analysis procedures. For example, a typical bias error in PIV results from the tracer particle that is used to seed the flow, since a heavy, large particle will not follow the flow as faithfully as a smaller, more nearly neutrally buoyant particle. The bias errors may be of a predictable magnitude, but their reduction requires a careful approach to each aspect of the PIV experiment and great care with the data analysis. Much effort has been expended in the analysis of these bias errors, and, for example, the error may be analyzed by theoretical analysis, conducting experiments under carefully controlled conditions or by generating synthetic PIV images of accurately known flow fields and subjecting these to a PIV analysis. The reader is referred to Raffel et al. (ref. 1), Lawson and Wu (ref. 2), and Soloff et al. (ref. 3) for examples of many such investigations.

Control of the bias errors and improvement of the overall accuracy of PIV depends upon having an appropriately seeded flow, the correct choice of hardware for the application, and the careful application of the image analysis algorithms. Another significant source of error lies with the calibration procedure, however, and the overall accuracy of the calibration depends on the quality of the calibration equipment and procedure as well as on the image analysis applied to the calibration images. A rigorous calibration for stereoscopic PIV is even more important than for two-component, single-camera PIV, as the difference in the two measured velocity fields is required, which therefore amplifies any errors.

The basic starting point for a calibration is a carefully manufactured calibration graticule that can be aligned precisely with and moved relative to the PIV laser light sheet to an accuracy of a few microns. Bjorkquist (ref. 4) performed an analysis of the effect of calibration-plate

offset and rotation with respect to the light sheet using synthetic PIV images, while Raffel et al. (ref. 1) assessed calibration-plate alignment errors using back projection of particle image recordings. Both studies showed that significant calibration errors could easily be the result of alignment errors equivalent to the laser sheet thickness.

Besides the physical alignment of the calibration plate with the laser light sheet, it is essential that the optical conditions for the calibration are identical to those of the actual PIV test. This poses significant challenges for PIV in flows where there are significant changes in the refractive index of the medium within the flow field, but also underlines how important it is that even seemingly trivial tasks such as imaging through windows in the testing apparatus should be done correctly. Indeed, it has been shown that a reduction of the image distortions of a stereo PIV test in a thick liquid medium can be achieved by placing a liquid-filled prism at the liquid/air interface (Prasad and Jensen (ref. 5)).

The current analysis considers wind tunnel PIV tests, where the PIV imaging is performed through wind tunnel windows. It is noted that calibration errors for this problem have been addressed by Soloff et al. (ref. 3), who showed that a volumetric calibration could adequately account for the distortion introduced by refraction through the window. Of concern here, however, is what errors may result from the use of a standard calibration procedure, where the calibration graticule is imaged in the plane of the laser sheet only. This is a widely used calibration procedure, and therefore an assessment of the error is important. It is shown that a non-uniform image distortion occurs across the field of view that is proportional to the window thickness, and it is then suggested that this error is manifested in the stereo reconstruction of the velocity field. For test facilities in which thick windows are present, the calibration procedure should be modified to reduce this effect by performing a volumetric calibration or by compensating for the refractive effect on the perspective angle.

REFRACTION EFFECT ON PERCEIVED PARTICLE DISPLACEMENT

Principle of Stereo PIV

It is firstly helpful to cover the basic theory for stereoscopic PIV, so that the importance of the camera view, or perspective angle, may be understood. Full details of the theory may be found in Raffel et al. (ref. 1), for example. Figure 1 shows the relative positions of the object plane and an off-axis camera, along with a definition of the coordinate system. Figure 2 shows a schematic diagram of light rays for an off-axis PIV test (e.g., for stereo PIV), in which only perspective effects are considered. A

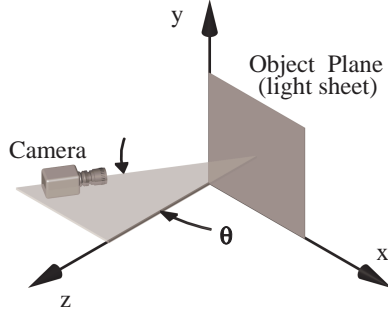


Figure 1: Coordinate system and camera position.

particle is recorded at two positions, 1 and 2. For convenience, location 1 is coincident with the center of the object plane. The imaging system records the projection $\Delta x'$ of the physical displacement $\Delta \underline{s}$ onto the object plane. The perspective error ϵ_x is the difference between the actual displacement in the object plane Δx and the apparent, projected displacement $\Delta x'$ such that

$$\Delta x' = \Delta x + \epsilon_x \quad (1)$$

$$\epsilon_x = \Delta z \tan \theta \quad (2)$$

where θ is the perspective angle in the x, z plane, which is the angle between the normal to the object plane and the line of the chief ray to the lens. The perspective angle varies along the light sheet, but it is usually close to the angle between the lens axis and the normal to the light sheet. Therefore, if there is a sizeable perspective angle, then the perspective error will be significant and the displacements Δx and Δz cannot be determined unless the flow is imaged from two separate views of known perspective angle. This leads to the so-called stereoscopic PIV, where the actual displacements can be determined by a stereo reconstruction of the two velocity fields by measuring the projected particle displacements from two different camera views. Regarding the two different camera views, L and R , of the same particle displacement in the flow, then there are two sets of measurements as follows:

$$\Delta x'_R = \Delta x + \Delta z \tan \theta_R \quad (3)$$

$$\Delta x'_L = \Delta x + \Delta z \tan \theta_L \quad (4)$$

Combining these two equations gives the actual particle displacements in the flow as

$$\begin{aligned} \Delta x &= \frac{\Delta x'_L \tan \theta_R - \Delta x'_R \tan \theta_L}{\tan \theta_R - \tan \theta_L} \\ \Delta z &= \frac{\Delta x'_R - \Delta x'_L}{\tan \theta_R - \tan \theta_L} \end{aligned} \quad (5)$$

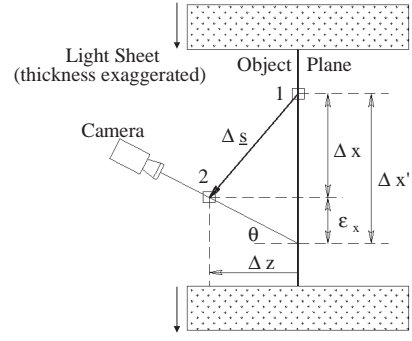


Figure 2: Notation for analyzing perspective error.

Strictly speaking, the perspective effect of an unintentional rotational displacement of the cameras in the y - z plane also has to be considered, but in most applications it is small. While the above two expressions provide the displacements in the x and z directions, the displacement in the y direction is simply

$$\Delta y = \frac{\Delta y'_L + \Delta y'_R}{2} \quad (6)$$

since there is no perspective effect in this direction.

Analysis of the Refractive Effect

Now the presence of a test section window will be taken into account. The refractive effect will be considered for two limiting cases: (a) the plane of the light sheet is parallel to the glass window; (b) the plane of the light sheet is orthogonal to the glass window. The latter configuration is commonly used in, for example, cross-flow measurements.

Object plane parallel to window— Figure 3 shows how the presence of a glass window can affect the light rays traveling from the particles to the image plane (i.e., the camera sensor). These rays are refracted as they pass through the window, which is idealized as a parallel sided block, and so they emerge from the window in a direction parallel to the incident rays. The back projections of the refracted rays onto the object plane show the physical distance between the incident and refracted rays. This displacement is given by

$$\delta = t(\tan i - \tan r) \quad (7)$$

where i and r are the incident and refraction angles of the ray and t is the thickness of the window. The refraction angle is related to the incident angle by Snell's law

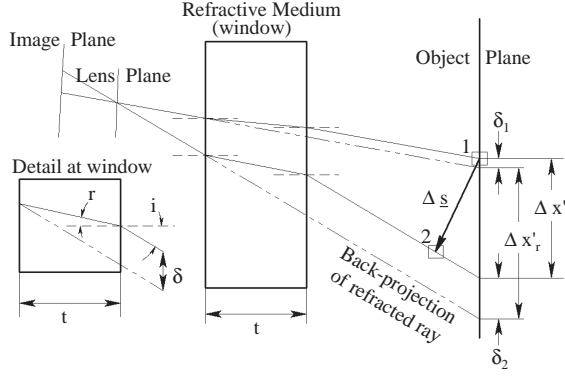


Figure 3: Effect of refraction on particle displacement.

$$n = \sin i / \sin r \quad (8)$$

where n is the refractive index and typically $n = 1.5$ for glass. It is helpful to consider what angles of incidence might be expected in a typical PIV setup. A 20-cm-wide area viewed normally from a distance of 2 m will yield incident angles at the window of no more than about 3° at the extremities of the viewing area. Typical stereo PIV bisecting angles for the case when both cameras are viewing the same side of the light sheet are in the region of 35° , so the refractive displacements in the stereo case will be at least 16 times greater than for viewing with a nominally zero perspective. The refracted particle displacement in the object plane is then given by

$$\Delta x'_r = \Delta x' + \delta_2 - \delta_1 \quad (9)$$

so that

$$\Delta x'_r = \Delta x' + t[(\tan i_2 - \tan i_1) - (\tan r_2 - \tan r_1)] \quad (10)$$

where the second term in the above expression represents the refractive contribution to the displacement. Referring to figure 4, the local incident angle at the window corresponding to a point at an ordinate p along the object is given by

$$\tan i = \frac{(l \sin \theta_c - c) + p}{l \cos \theta_c} \quad (11)$$

where l is the distance from the lens to the middle of the object and $2c$ is the length of the object (i.e., the field of view). The refraction angle is straightforward to calculate from Snell's law. The incident and refracted angles for an in-plane particle displacement Δx (equivalent to Δp) can also be calculated, and hence the difference between the refracted particle displacement and the actual particle displacement can be found.

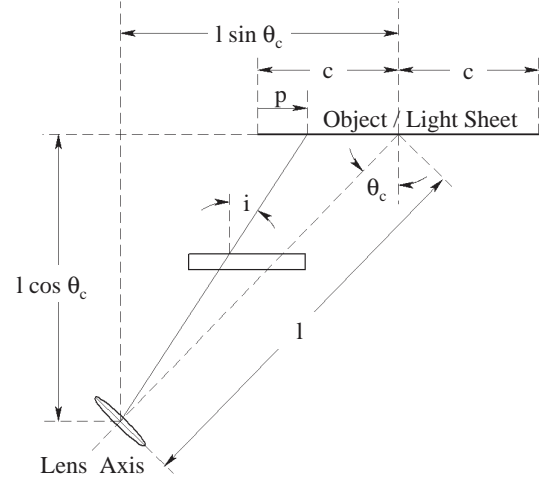


Figure 4: Variable definitions with the light sheet parallel to the window.

Figure 5 shows the relative difference between the actual particle displacement and the refracted particle displacement for an example stereo view across a range of perspective angles, assuming a field of view of $2c = 0.2$ m, a viewing distance of $l = 2$ m, an in-plane particle displacement of $\Delta x = 2$ mm, and a window thickness of $t = 38$ mm. The difference (based on equations 8, 10, and 11) represents an error and is presented as the ratio of the refractive error (difference between refracted and actual displacements) to the actual displacement. The error is the least at $p/c = 0$, where the incident angles to the window are smallest.

It is easily shown that the error itself is not strongly dependent upon the particle displacement, and the analysis above indicates that the significant sources of the refractive error are the perspective angle θ and the thickness t of the window. Since the error is directly proportional to the window thickness t , its effect has not been shown on the plot. The error increases with distance along the object since the angle of incidence increases in the direction p , and the error is positive for the same reason.

The effects of field of view and viewing distance are simply to change the range of variation of the error across the field of view as the incidence angle at the window changes. In stereo PIV the light sheet is viewed from two directions, and the perspective angles for the two cameras are frequently set to be approximately equal in magnitude. In the case where the two cameras view the flow from the same side of the light sheet, the refractive errors at the extremities of the field of view will be different for the two cameras. Conversely, in the case where the cameras view the flow from opposite sides of the light sheet, the refractive errors for the two cameras will be

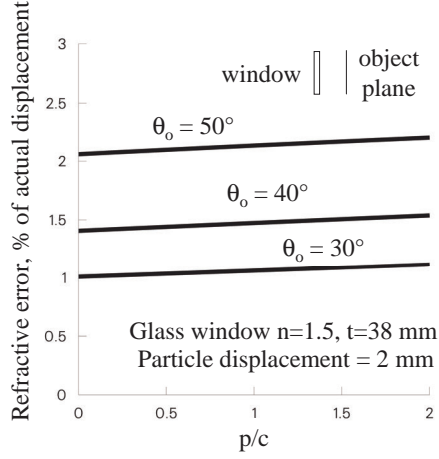


Figure 5: Particle displacement error with the light sheet parallel to the window.

approximately the same. The particular refraction error under discussion here will not, of course, appear if the calibration is performed through the window, and the results shown on the figure show that if the calibration is not performed while imaging through the window a significant error may be the result.

Object plane orthogonal to window– Figure 6 shows the plane of the light sheet as being orthogonal to the window, and how the refraction of the light ray affects the position of the particle in the plane of the light sheet. The coordinate system for calculating the angle of incidence at the glass window is also shown. The difference between this light-sheet orientation and the previous case is that the shift of the light ray in the direction of the normal to the glass window is important (i.e., δ_N , as indicated). This shift is related to the refractive shift δ by the expression

$$\delta_N = \delta / \tan i \quad (12)$$

and so

$$\delta_N = t(1 - \tan r / \tan i) \quad (13)$$

The angle of incidence of a light ray at the window in this case is given by

$$\tan i = \frac{l \cos \theta_c}{(l \sin \theta_c - c) + p} \quad (14)$$

and the rest of the analysis is the same as the parallel-window case. Figure 7 shows the error associated with refraction for the case with the light sheet orthogonal to the glass window and for the same basic conditions as

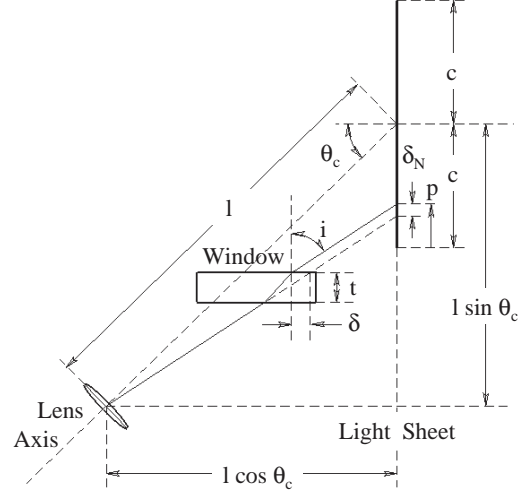


Figure 6: Variable definitions with the light sheet orthogonal to the window.

for the parallel-window case. The key distinction here is that the angle of incidence decreases as p increases, which means that the difference between the perceived and actual displacements is negative and its magnitude decreases with increasing p . The magnitude of the error is significantly less than for the parallel orientation, primarily because the refractive shift δ_N is almost uniform along the light sheet.

REFRACTION EFFECT ON THE CALIBRATION PROCEDURE

Recall from the analysis in the previous section that the actual particle displacements for stereo PIV are given by

$$\begin{aligned} \Delta x &= \frac{\Delta x'_L \tan \theta_R - \Delta x'_R \tan \theta_L}{\tan \theta_R - \tan \theta_L} \\ \Delta y &= \frac{\Delta y'_L + \Delta y'_R}{2} \\ \Delta z &= \frac{\Delta x'_R + \Delta x'_L}{\tan \theta_R - \tan \theta_L} \end{aligned} \quad (15)$$

The PIV analysis and calibration provide the projected displacements $(\Delta x'_L, \Delta y'_L)$ and $(\Delta x'_R, \Delta y'_R)$. The perspective angles θ_L and θ_R are used for the stereo reconstruction to find the three velocity components, and these must be known to high accuracy for each camera as otherwise they will introduce an error into the stereo reconstruction. They may be determined by either (a) inferring them from the optical magnification of the system and using the system geometry for the Scheimpflug condition, or (b) performing geometric measurement of the object, lens, and cameras positions in space. An analysis of the refractive

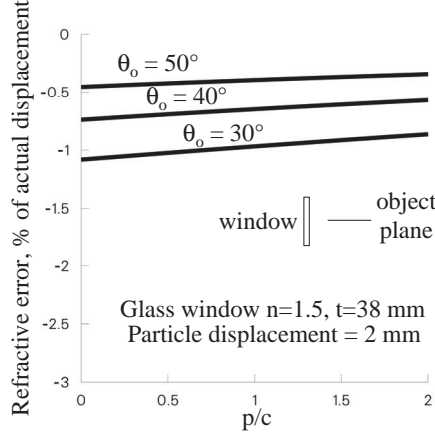


Figure 7: Particle displacement error with the light sheet orthogonal to the window.

error on these two methods is provided in the following sections.

Determining the Perspective Angle from Optical Magnification and the Scheimpflug Condition

The Scheimpflug condition is met when the object, lens, and image planes are coincident at some point in space. This provides the basis for the geometry indicated in figure 8. From the geometry, the perspective angle θ at an arbitrary point p along the object plane can be determined from

$$\tan \theta = \frac{d_o \sin \theta_o - p_o + p}{d_o \cos \theta_o} \quad (16)$$

It is the local perspective angle that should be used for the stereo reconstruction. The important parameters to be found are those relating to where the lens axis crosses the object plane (θ_o , p_o , and d_o). Following the analysis of Prasad and Jensen (ref. 5), it is convenient to place a coordinate system (λ, q) at the center of the lens. The λ coordinate defines the position along the lens plane, and the q coordinate defines the distance of a point away from the lens plane in the direction of the object. The parameter d_o is the distance to the object plane from the lens plane measured along the lens axis. The coordinates of a point on the object plane are given by

$$q_{obj} = d_o - \lambda \tan \theta \quad (17)$$

where the *obj* suffix indicates a point on the object plane. With the system correctly arranged in the Scheimpflug condition, all points on the object plane are brought into

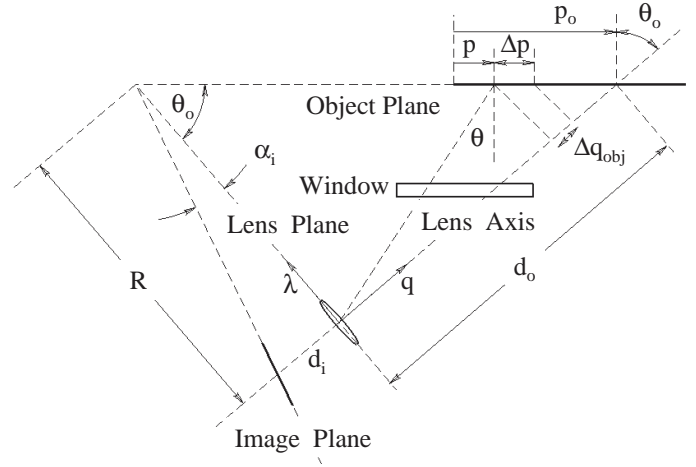


Figure 8: Scheimpflug condition and coordinate system relative to the lens.

focus on the image plane by the lens. If q_{im} is the q coordinate position of the image of the object on the image plane, then the local magnification is given by

$$M = -\frac{q_{im}}{q_{obj}} \quad (18)$$

The negative sign in the above expression is necessary because q_{im} is negative. When the image is in focus, the q coordinates of the object and image are related to the focal length f of the lens according to

$$\frac{1}{f} = \frac{1}{q_{obj}} - \frac{1}{q_{im}} \quad (19)$$

The above two equations can be combined to give

$$q_{obj} = f \frac{1+M}{M} \quad (20)$$

If the object with a known set of overall physical dimensions is imaged, and the dimensions of the image can be measured accurately, then the local magnification M for any position p along the object can be determined, and hence the object distance d_{obj} can be calculated. The coordinate λ can be determined after additional consideration of the Scheimpflug geometry, but if it is considered that in a PIV calibration the coordinate p along the object is known at a number of locations, then the perspective angle θ_o may be determined from

$$\Delta q_{obj} = \Delta p \sin \theta_o \quad (21)$$

where Δq_{obj} is the change in the object coordinate q_{obj} for a known increase Δp in the object coordinate p along the object plane (recall figure 8). It is therefore possible to find θ_o if the magnification can be determined, which is

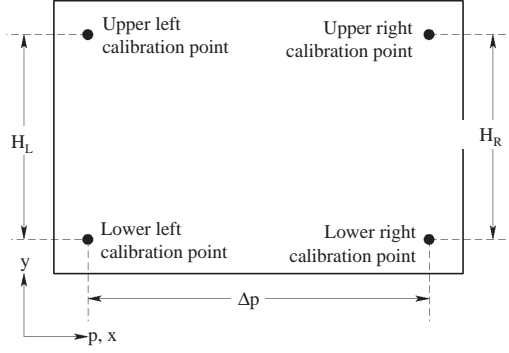


Figure 9: Calibration grid notation with object coordinate system.

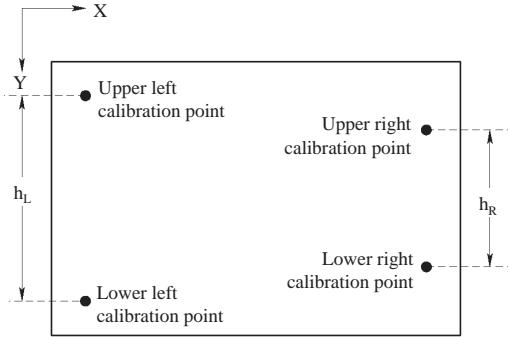


Figure 10: Image of calibration grid with image coordinate system.

possible in a digital system if the pixel size of the sensor is known, or in a film system by direct measurement of the film recording. With reference to figures 9 and 10, which show a simple calibration grid and its image respectively, this is done as follows:

1. Measure the object heights H_L and H_R at the left and right hand sides of the calibration grid.
2. Measure the horizontal distance Δp between the left and right sides of the grid.
3. Determine the pixel positions (X, Y) of the images of the objects at the left and right hand sides of the camera sensor. The image heights ΔY_L and ΔY_R in pixels are required. Then convert these to the physical image heights h_L and h_R by multiplying the pixel heights by the pixel size.
4. The magnification is $M = h/H$, so the magnifications at the left and right hand sides of the image

should be calculated.

5. Calculate the object distances q_{obj} for the left and right hand sides of the image (using equation 20), and hence determine the change in lens plane to object distance Δq_{obj} between the two ends of the object, taking care to respect the sign of Δp .
6. Calculate θ_o (using equation 21).

Figure 11 shows an image of a calibration grid. The dimensions of the extreme left and right hand sides of the image of the calibration grid are indicated (the pixel size in this case is $6.45 \mu\text{m}$), along with the object distance between the left and right hand sides. The left hand magnification is $M_L = 0.0964$, and the right hand magnification is $M_R = 0.0891$, which for the lens focal length of $f = 135 \text{ mm}$ gives (using equation 20) object distances of $q_{obj,L} = 1535 \text{ mm}$ and $q_{obj,R} = 1650 \text{ mm}$ respectively. According to equation 21, this gives the perspective angle as $\theta_o = 55.23^\circ$. The above procedure could of course be carried out for a larger number of points across the calibration grid, in which case a linear fit between Δq_{obj} and Δp for the set of calibration points would be obtained. To find the local perspective angle, the distance d_o and the coordinate p_o have to be found. To do this, the above analysis can be repeated to find the image plane coordinate q_{im} and the physical distance along the sensor, which provides the angle α_i between the image plane and the lens plane from exactly the same theory as above but applied to the image plane. The distance R between the lens center and the Scheimpflug coincident point can be determined since

$$\begin{aligned} d_o &= R \tan \theta_o \\ d_i &= R \tan \alpha_i \end{aligned} \quad (22)$$

where d_i is the distance of the image plane from the lens center, measured along the lens axis, and by definition

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \quad (23)$$

This then permits d_o to be calculated and effectively locates the origin of the (λ, q) coordinate system relative to the object plane, thereby allowing p_o to be found. For the present example, and with reference to data shown in figure 11, the above theory gives $q_{im,L} = -148.01 \text{ mm}$ and $q_{im,R} = -147.03 \text{ mm}$ (so $\Delta q_{im} = 0.987 \text{ mm}$), and for the calibration grid image width of 7.721 mm the remaining Scheimpflug angle is $\alpha_i = 7.34^\circ$. Note that calculation precision is essential here, so if the reader wishes to check the calculations, the raw data on figure 11 should be used to calculate the object plane parameters again to preserve

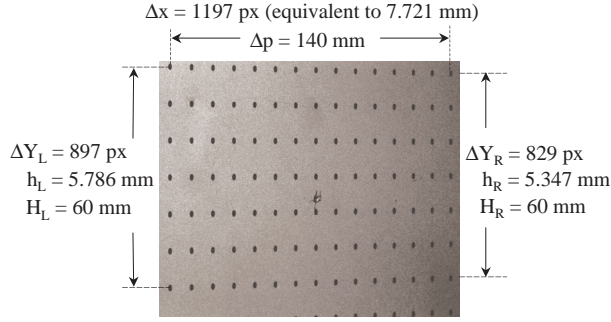


Figure 11: Image of a typical calibration grid.

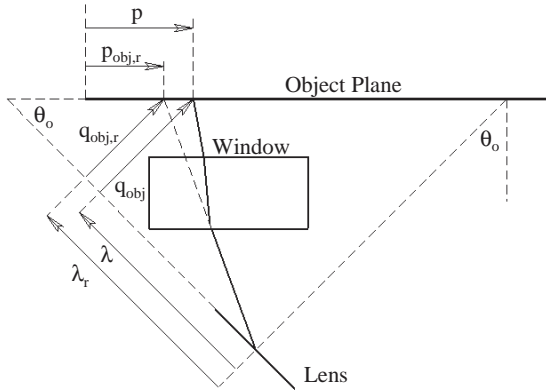


Figure 12: Virtual coordinate system for lens and object.

precision, rather than simply using the object plane values given above. Thus $R = 1141$ mm giving $d_o = 1643$ mm and $p_o = 112$ mm.

The above analysis procedure requires that the calibration object size be known, the image dimensions be measured accurately, and the Scheimpflug condition be satisfied, and that each of these are achieved with care. It now becomes clear how the refraction through the window can affect the calculation of the perspective angle, as the presence of the refracting window has the effect of shifting the position of the object. Figure 12 illustrates this, where the light ray from point (λ, q_{obj}) actually arrives at the lens as if it were from the shifted virtual location $(\lambda_r, q_{obj,r})$ with the glass window removed. The virtual location is shifted relative to the actual location by the refraction. When finding the perspective angle the refracted object positions must be used, so strictly speaking,

$$\Delta q_{obj,r} = \Delta p_r \sin \theta_o \quad (24)$$

where $\Delta q_{obj,r}$ is determined from the above described image analysis without the need for any refraction correction. To find Δp_r , the back projection of the actual object displacement Δp has to be found, and this is achieved using the same analysis as for the particle displacement shift above. This could be done iteratively by initially setting $\Delta p_r = \Delta p$, finding θ_o , applying a refractive correction to find Δp_r , and repeating the calculation until a suitable convergence is achieved. By applying the back projection, the refractive error on the distance d_o is automatically accounted for. Local refractive corrections would have to be applied to find the local perspective angle, and the procedure for this would be similar to above.

A typical calibration graticule consists of a grid of points (a dot pattern or a line pattern), and given that the angle θ_o is to be determined from the distance between the calibration points, the effect of the refractive shift is the same as the effect of refraction on the particle displacement for PIV. A typical grid pattern pitch is 5 mm, and figure 13 shows the refractive error in the quantity $\Delta p = 5$ mm for the window thickness of 38 mm for the parallel and orthogonal orientations for perspective angles of 35° and 55° . The choice of these two perspective angles is useful because in the parallel case the perspective angle of 35° results in a perspective angle of 55° for the orthogonal light sheet/window orientation if the window orientation only was changed to the orthogonal configuration, leaving unchanged the camera and laser sheet positions. As with the PIV displacement error, the calibration stretch is of significantly smaller magnitude in the orthogonal orientation case. The error of 1.2% corresponds to an angular error of about 0.7° from a basic perspective of 35° , and given that $\tan \theta$ is required in the stereo reconstruction, the significance of this would be approximately 3% error in the out-of-plane velocity estimation. For the orthogonal orientation the error is less than half that of the parallel case.

Determining the Perspective Angle from the System Geometry

Figure 14 shows the geometry of the one half of the stereo PIV system for finding the perspective angle in the parallel configuration. The local perspective angle at an arbitrary point along the object is found by the same basic method as described in the previous section, except here θ_o and p_o are to be measured directly. It is difficult to ascertain exactly where the lens axis lies, and it is easiest in practice to measure the dimensions to the center of the object. Therefore, it is assumed in the remainder of this analysis that the angle θ_c is to be determined by

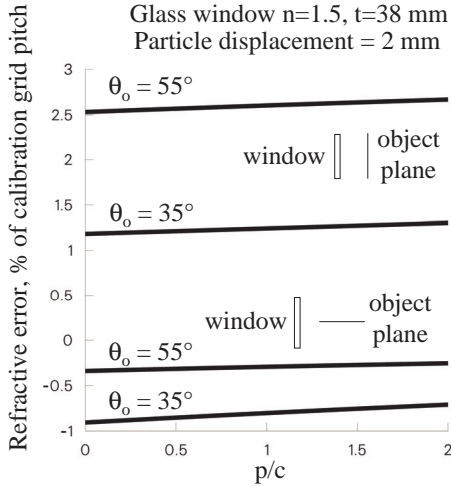


Figure 13: Calibration pitch error due to refraction.

direct measurement. The geometric center of the calibration object is straightforward to define, but the center of the lens is not, so the accuracy of the angle θ_c will improve for large imaging distances. However, with the window present the lens axis on the lens side of the window is shifted relative to the object side, so to find the perspective angle the shift must be accounted for and so

$$\tan \theta_c = \sigma_r / a \quad (25)$$

For a window thickness of 38 mm and a central perspective angle of 35° , the shift is 10 mm in the direction indicated. It is clearly advantageous in this case to have as large a viewing distance as possible. For the viewing distance of 2 m, the error caused by the refractive shift is 0.2° . This is much smaller than for the previous case, but assumes that the geometry can be measured accurately in the first place.

DISCUSSION OF THEORETICAL ANALYSIS

The effect of the refracting window is to introduce a stretching across the field of view, and in the case of stereo PIV, where the perspective is nominally in one direction only, the stretching will affect only one component of the projected displacement. If the calibration is performed with the window in place, then this stretching is automatically accounted for in the PIV analysis and theoretically no additional error arises (see below for a discussion of the stereo reconstruction error). However, if the calibration is not done with the window in place, or if the window is somehow altered between calibration and execution of the PIV experiment, then an error

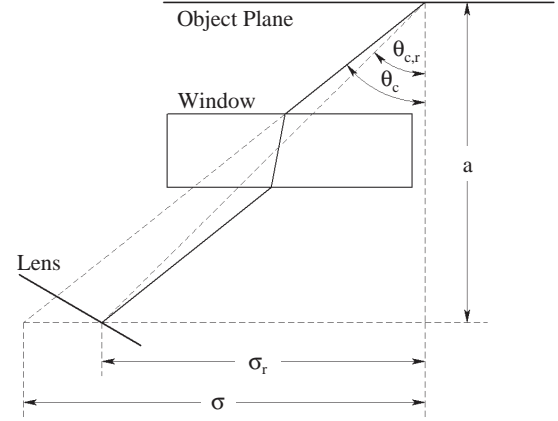


Figure 14: Projected and actual distances from lens to object.

is introduced into the PIV analysis. This error is proportional to the window thickness and depends upon the nominal perspective angle, and in a severe enough case the magnitude of the error could exceed 2% of the actual projected displacement. For the case when the perspective is in one plane only (single-axis Scheimpflug), only one component of the projected particle displacement is affected, and this means that derived quantities such as vorticity and divergence would be affected. The basic measurement error might seem to be small, but in PIV there are many sources of error that are compounded, and so the calibration should be performed in an optical environment that is as close to the actual experimental as possible.

It is clear from the analysis that if a “standard” calibration using a calibration plate aligned with the PIV light sheet is performed with the cameras viewing through the window, then no systematic error in the projected particle displacements is expected to arise. However, the velocity field has to be reconstructed from the set of projected particle displacements, and a source of error appears to arise from the manner in which the perspective angle is calculated. This error may be significant, and could be corrected for by accounting for the refraction effect when the perspective angle is calculated. However, given the prior experimental evidence of the performance of volumetric calibration methods (Soloff et al. (ref. 3)), it is perhaps better to use such a calibration procedure to start with.

The benefit of applying a refractive-error correction in the stereo reconstruction would arise in a retrospective analysis of an existing set of data, where the expense of repeating the experiment would not be worth the recovery of a few percent of accuracy. In this respect

the evaluation of the perspective angle is a potentially significant source of error in the analysis of stereo PIV data anyway. The method described earlier for determining the perspective angle from optical magnification and the Scheimpflug condition, where no measurement of the camera and calibration geometry is required, assumes that the Scheimpflug condition has been satisfied, and this requires that the image be judged to be correctly in focus across the field of view.

In the analysis presented for determining the perspective angle from optical magnification and the Scheimpflug condition, the change in image distance Δq_{im} was less than 1 mm across the image sensor distance of about 7 mm, meaning that great accuracy is required in the calculation of the angle α_i . This implies that the Scheimpflug condition has to be satisfied with great precision. Furthermore, the locations of the calibration grid points in the image need to be measured with great accuracy in order that all the various geometric parameters in the calculation are computed accurately. For example, even assuming that the Scheimpflug condition was precisely met for the case shown in figure 11, if the vertical locations of the centers of the dots on the image are subject to an error of only one pixel for each side, the perspective angle may be in error by $\Delta\theta_o = 3^\circ$, equivalent to an error in $\tan\theta_o$ of 10% for $\theta_o = 55.2^\circ$. Equivalent pixel errors produce a smaller error in θ_o as the perspective angle falls. The error in $\tan\theta_o$ is about 6% for $\theta_o = 35^\circ$, and the error in the w component of velocity is of this magnitude.

In the context of the current study, the effect of the refractive error with the standard calibration technique is to compound an error that is potentially significant anyway, and it is proposed here on the basis of this analysis that all stereo PIV experiments be conducted using a volumetric calibration, where the calibration is performed by imaging the calibration grid in a number of closely spaced (x, y) planes. Using this calibration method would act to emphasize the care required in calibration for stereo PIV. In this calibration method the stereo reconstruction does not require the use of the perspective angle (see Soloff et al. (ref. 3)), as the (u, v, w) flow field is built up from the stereo PIV data using the three-dimensional calibration. This is intrinsically more robust than the standard calibration as it does not make any assumptions about the focussing arrangement and does not require measurement of the relative positions of the camera and light sheet.

Indeed the studies of Lawson and Wu (ref. 2) and Soloff et al. (ref. 3) both employed the volumetric calibration technique, but used different camera focussing arrangements. Lawson and Wu (ref. 2) used a simple arrangement with no lens/camera offset and the camera tilted with respect to the calibration plane. Sharp focus was achieved using a high lens f-number. On the other hand, Soloff et al. (ref. 3) performed experiments with the

camera sensor, lens, and object planes all parallel to each other, achieving field-of-view overlap by using a translational lens shift. This ensures good focussing at low f-number, but the joint field of view is relatively small. Note that neither of these studies used the Scheimpflug focussing arrangement. The basic experimental approach applied in these two studies was otherwise similar, but for mathematical representation of the calibration geometry in the image coordinates, Lawson and Wu (ref. 2) used a bicubic spline interpolation while Soloff et al. (ref. 3) used a least-squares polynomial, with cubic dependence on the in-plane displacements and a quadratic dependence on the out-of-plane displacement. In neither case was any assumption made about the camera, lens, and calibration grid setup. In addition, as the volumetric calibration method can account for any combination of $(\Delta x, \Delta y, \Delta z)$ particle displacement (within the limits of the calibration) then the technique ought to be less sensitive to a slight misalignment of the laser sheet and calibration grid, which is known to be a significant source of error in stereo PIV (Bjorkquist (ref. 4), Raffel et al. (ref. 1)). Bjorkquist (ref. 4) deliberately applied rotation and alignment displacements in an experimental simulation of light sheet/calibration plane misalignment in PIV, but was able to correct for the target misalignment using data from the volumetric calibration, which would not be possible using the standard calibration.

Given the significance of the perspective angle in the stereo reconstruction when the standard calibration is used, stereo PIV experiments in the case where the perspective angle is large in both the (x, z) and (y, z) planes suffer an even larger source of potential error than in the case analyzed here, even without the presence of the refracting window, and this is because all three velocity components require stereo reconstruction. The volumetric calibration procedure used in this case would introduce no additional error when compared to the standard procedure. Although the analysis performed in support of the current study has indicated that the standard calibration procedure leads to errors, there is no suggestion that the technique should be abandoned altogether. It is merely reinforced that the various assumptions made in the analysis of the calibration images require that great care be taken in executing the calibration procedure, otherwise the stereo perspective angle may be in error. However, perhaps this degree of care is best directed, instead, at performing the volumetric calibration. The analysis of the effect of the refracting window has therefore gained additional value in that it has highlighted how the assumptions used in the standard calibration can lead to a significant contribution to both the random and bias errors referred to earlier.

EXPERIMENT TO VERIFY THE REFRACTION EFFECT DUE TO A GLASS WINDOW

The effect of the refracting window is to introduce a non-uniform distortion across the field of view, and the calibration will account for this if (and only if) sufficient data points are taken across the field of view to allow an adequate curve fit to the distortion. Provided the calibration was performed with the same optical path as the actual experiment, there is no reason to suppose that the presence of a thick, refracting window will have an adverse effect on the PIV as far as determining the projected particle displacements $\Delta x'$ is concerned. The stereo reconstruction may well suffer, however, and therefore the accuracy of the three velocity components may be degraded by an incomplete treatment by a disregard for the presence of the window. This is because the perspective angle θ plays a fundamental role in the stereo reconstruction as follows:

$$\begin{aligned}\Delta x &= \frac{\Delta x'_L \tan \theta_L - \Delta x'_R \tan \theta_R}{\tan \theta_L - \tan \theta_R} \\ \Delta z &= \frac{\Delta x'_L - \Delta x'_R}{\tan \theta_L - \tan \theta_R}\end{aligned}\quad (26)$$

For the displacement Δy the perspective effect is often very small, so the following expression usually suffices:

$$\Delta y = \frac{\Delta y'_L + \Delta y'_R}{2}\quad (27)$$

A set of experiments to investigate the importance of the refraction by the glass window is as follows. No laser is required, but some means of representing PIV particles in the object plane must be used. This can be done using, for example, a photograph printed on a suitable medium of a pattern that will produce an image representative of particles in an actual PIV experiment, and effectively simulates an illuminated slice of the flow. This must be aligned precisely with the calibration target, as in an actual PIV experiment.

1. Set up the stereo PIV with both cameras in the same horizontal plane and with both cameras on the same side of the simulated sheet of particles to investigate the refractive effect with the light sheet parallel to the window.
2. Without glass window:
 - (a) Perform calibration.
 - (b) Move synthetic particle pattern in precisely measured sets of $(\Delta x, \Delta y, \Delta z)$ steps and obtain images. It is essential that steps in the x , y , or z directions only are performed in addition

to steps based on combinations of those directions, as this will permit the effects of stereo reconstruction to be isolated. Note that relatively few measurements of Δy displacement are required, as in this arrangement the perspective effect is small.

- (c) Perform PIV analysis of data, and compare results with actual displacements. The analysis of the PIV results should proceed carefully.

3. Repeat procedure 2 with glass window.
4. Repeat entire procedure from 2 with different stereo separation angles.
5. Repeat entire procedure from 1 with a simulated particle field and window in the orthogonal orientation.

A data analysis to isolate calibration distortion from stereo reconstruction error would then need to consider at least the following points:

1. For Δy displacements only (i.e., with other displacements zero), compare actual data with PIV data with and without the window. This will permit the theoretically small refraction distortion in the y direction to be assessed.
2. For Δz displacements with a constant Δx displacement, compare Δz displacements from the PIV analysis with the actual values with and without the window. The $\Delta x'$ values (i.e., PIV analysis data before the stereo reconstruction has been performed) should also be taken, so that the calibration distortion error can be isolated from the stereo reconstruction error (i.e., the contribution due to error in perspective angle θ).
3. For Δx displacements only (i.e., other displacements zero) compare $\Delta x'$ displacements from PIV with the actual Δx displacement with and without the window. This will allow the effect of the perspective angle (and hence a larger refractive effect) to be seen in the refractive distortion on the x direction.

An alternative to translating the simulated particle field by Δy would be to rotate the field about its z axis instead. This would simulate a rotational flow field with uniform vorticity (zero vorticity in this case) and would show how the process is affected when there are displacement gradients in the x and y directions. For the experiment just described, and given that only the Δx displacement would be significantly affected by the presence of the window, any error in the calculated vorticity could be attributed to a displacement gradient that is analogous to the $\partial u / \partial y$ contribution to vorticity. In other words, a truly rotational motion of the simulated particle field would result in an

erroneous vorticity with a magnitude that depends on the refractive error caused by the window.

CONCLUSIONS

An analysis of the error in stereo PIV data due to imaging through a glass window has been performed. When a standard calibration based on aligning a calibration plate with the PIV light sheet is performed, the error in the PIV analysis arises when the stereo reconstruction is performed as a result of an incorrect calculation or measurement of the perspective angle. It is indicated how the stereo reconstruction error may be isolated using an idealized stereo PIV experiment. The perspective angle measurement may be significantly in error anyway, and it is suggested that the volumetric calibration method be used to avoid this source of systematic error.

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